Question 1

Derivative of $f(x) = \dfrac{5x^2(3-2x^2)}{(1+x)^3}$

= Quotient rule

$$\frac{(1+x)^3 \frac{d}{dx}\left(5x^2(3-2x^2)\right) - 5x^2(3-2x^2)\frac{d}{dx}(1+x)^3}{(1+x)^4}$$

$$= (1+x)^3 \cdot 5x^2 \frac{d}{dx}(3-2x^2) + 3-2x^2 \frac{d}{dx}(5x^2) -$$

$$\frac{5x^2(3-2x^2)(3(1+x)^2 \cdot 1}{(1+x)^4}$$

$$= \frac{\{(1+x)^3 \cdot 5x^2(-4x) + 3-2x^2 \cdot 10x\} - 5x^2(3-2x^2) \cdot 3(1+x)^2}{(1+x)^4}$$

$$= \frac{(1+x)^3 \cdot -10x^3 + 30x - 20x^3 - 15x^2 - 10x^4 \cdot 3(1+x)^2}{(1+x)^4}$$

$$\frac{(1+x)^3 - 30x^3 + 30x - 15x^2 - 10x^4 \cdot 3(1+x)^2}{(1+x)^4}$$

$$= \frac{\cancel{(1+x)^3} - 30x^4(1+x)^2 - 30x^3 - 15x^2 + 30x}{\cancel{(1+x)^3}(1+x)}$$

$$= \frac{30x - 30x^4\cancel{(1+x)}(1+x) - 30x^3 - 15x^2}{\cancel{1+x}}$$

$$= 30x - 30x^4 - 30x^5 - 30x^3 + 15x^2$$

$$f'(x) = \frac{30x^5 + 30x^4 + 30x^3 + 15x^2 - 30x}{15}$$

$$f'(x) = 2x^5 + 2x^4 + 2x^3 - x^2 - 2x$$

Question 2

Derivative of $f(x) = (2x+4)^4 (x^5+x-3)^3$

we use the product rule

$$\frac{df}{dx} = (2x+4)^4 \frac{d}{dx}(x^5+x-3)^3 + (x^5+x-3)^3 \frac{d}{dx}(2x+4)^4$$

$$= (2x+4)^4 \cdot 3(x^5+x-3)^2 \frac{d}{dx}(x^5+x-3) + (x^5+x-3)$$
$$\frac{d}{dx}(2x+4) \cdot 4(2x+4)^3 .$$

$$= (2x+4)^4 \cdot 3(x^5+x-3)^2 \cdot 5x^4+1 + x^5+x-3 \cdot 2\cancel{x+4} \cdot 4(2x+4)^3$$

$$= 3(2x+4)^4 (x^5+x-3)^2 (5x^4+1) + 4(2x+4)^3 (x^5+x-3)^3 \cdot 2$$

Noticing that each term has the common factor $2(2x+4)^3 (x^5+x-3)^2$ we could factor it out

$$= 2(2x+4)^3 (x^5+x-3)^2 (32x^5+60x^4+8x+6)$$

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Question 3

Write expression for $h'(x)$ given

$$h(x) = p(x)\ q(x)\ r(x)\ s(x).$$

$$h(x) = p(x) \cdot q(x) \cdot r(x) \cdot s(x)$$

$$h'(x) = p'(x)\ q'(x) \cdot r'(x) \cdot s'(x)$$

Since $h'(x)$ is expressed as various functions.

Question 4

$f(x) = ax^3 + bx^2 + cx + d$ Determine abc and d so that $f'''(x) = 11$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f'''(x) = 6a$

but $f'''(x) = 11$

$6a = 11$

$a = \frac{11}{6}$

$f''(x) = 6ax + 2b$

$f''(x) = \cancel{6}\left(\frac{11}{\cancel{6}}\right)x + 2b$

$f''(x) = 11x + 2b$

$b = -\frac{11}{2}x$

$-c = 3ax^2 + 2bx$

$-c = 3\left(\frac{11}{6}\right)x^2 + 2\left(-\frac{11}{2}x\right)x$

$-c = \frac{11}{2}x^2 - 11x^2$

$-c = -\frac{11}{2}x^2$

$c = \frac{11}{2}x^2$

$-d = ax^3 + bx^2 + cx -$

$-d = \frac{11}{6}x^2 + \left(\frac{11}{2}x\right)x^2 + \left(\frac{11}{2}x^2\right)x$

$-d = \frac{11}{6}x^2 - \frac{11}{2}x^3 + \frac{11}{2}x^3$

$d = \frac{11}{6}x^2$

Question 5

If $f(3) = 7$ and $f'(3) = -3$ determine the exact value of $g'(3)$ where $g(x) = 2x^2 \cdot f(x)$

$$g'(x) = 4x \cdot f'(x).$$

$$g'(3) = (4 \times 3) \cdot f'(3).$$

$$g'(3) = 12 \cdot (-3).$$

$$g'(3) = \underline{\underline{-36}}$$

Question 6

For which values of x are the slopes of the tangent to $f(x) = \frac{x+3}{x-3}$ and $g(x) = \frac{x+2}{x-1}$ equal

$$f'(x) = \left(\frac{x+3}{x-3}\right)'$$

$$= \frac{(x+3)'(x-3) - (x+3)(x-1)'}{(x-3)^2}$$

$$= \frac{(x-3) - (x+3)}{(x-3)^2}$$

$$\cancel{x} - 3 - \cancel{x} - 3 = \frac{-6}{(x-3)^2}$$

$$g(x) = \frac{x+2}{x-1}$$

$$g'(x) = \left(\frac{x+2}{(x-1)}\right)'$$

$$= \frac{(x+2)'(x-1) - (x+2)(x-1)'}{(x+1)^2}$$

$$= \frac{(x-1) - (x+2)}{(x-1)^2}$$

$$= \frac{\cancel{x} - 1 - \cancel{x} - 2}{(x-1)^2} = \frac{-3}{(x-1)^2}.$$

$$\frac{-6}{(x-3)^2} = \frac{-3}{(x-1)^2}$$

$$= -6(x-1)^2 = -3(x-3)^2$$

$$= -6(x^2 - x - x + 1) = -3(x^2 - 3x - 3x + 9)$$

$$= -6(x^2 - 2x + 1) = -3(x^2 - 6x + 9)$$

$$= -6x^2 + 12x - 6 = -3x^2 + 18x - 27.$$

$$= 3x^2 + 6x - 21 = 0$$

$$x^2 + 2x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$x = \frac{-2 \pm \sqrt{4 + 28}}{2}.$$

$$= \frac{-2 \pm 5.65}{2}.$$

$$x = 3.828 \text{ or } x = -3.828.$$

Question 7.

When the derivative is negative, the graph of the derivative ~~lies in~~ is below the x-axis

Example:

① $f(x) = -x^2$ Negative $(0, \infty)$

② $f(x) = -x^2 + 2x$ $(1, \infty)$

③ $f(x) = x^2 - 36$ $(-\infty, 0)$

Question 8)

$f(x) = \frac{2(x^2+3)}{1-x^3}$ obtain $f'(x) = \frac{4x}{-3x^2}$

✓ Steve find the derivative without considering any derivative rule instead Steve find the derivative of the numerator and denominator directly

✓ The rule of Quotient Rule should be used that is

$$\frac{2x^2+6}{(1-x^3)}$$

$$f'(x) = \frac{1-x^3 \frac{d}{dx}(2x^2+6) - 2x^2+6 \frac{d}{dx}(1-x^3)}{(1-x^3)^2}$$

$$= \frac{(1-x^3)(4x) - 2x^2+6(-3x^2)}{(1-x^3)^2}$$

$$= \frac{4x - 2x^2+6 - 3x^2}{(1-x^3)}$$

Question 9

$$R(x) = \frac{15x - x^2}{x^2+15}$$

(a) Determine the rate of change for the sales of 800 items and 2400 items.

$$\text{Rate of Change} = \lim_{x \to x_2} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$R(800) = \frac{15(800) - (800)^2}{(800)^2 + 15}$$

$$R(800) = \frac{12000 - 640000}{640000 + 15}$$

$$R(800) = \frac{-628{,}000}{640015}$$

$$R(800) = -0.98$$

$$R(2400) = \frac{36000 - 5760000}{5760000 + 15}$$

$$= -0.9937.$$

(b) Compare the values:
in 2400 items there is ~~great~~ slight degree in Revenue compared to 800 items.

(c) Number of Items such that Rate of Change $= \$0$

$$0 = \frac{15x - x^2}{x^2+15}$$

$$x^2 + 15 = 15x - x^2.$$

$2x^2 + 15 = 15x$

$= \cancel{2}x^2 - 15x + 15 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2b}$$

$$= \frac{15 \pm \sqrt{225 - 120}}{4}$$

$$\frac{15 \pm \sqrt{105}}{4} \Rightarrow \frac{15 \pm 10.25}{4}$$

$$= \frac{25.25}{4} \simeq 6.3$$

$= \underline{\underline{6 \text{ items}}}.$

d) Revenue for the value found in c.

$$f'(x) = \frac{(15x - x^2)(x^2+15)' - (15x - x^2)'(x^2+15)}{(x^2+15^2)}$$

$$= \frac{2x(15x - x^2) - 15 + 2x}{x^2 + 15}$$

$$\text{Revenue} = \frac{2(6)(15(6) - (6^2)) - 15 + 12}{36 + 15} = \frac{648 - 15 + 12}{51} \quad \cancel{12(54)}$$

Revenue $= \$ \; 12.64$

Question 10

Sells 120 DVD's per week at \$24 each.

each \$ 0.75 decrease in Price, 5 additional CD's will be sold per week

(a)

sold $= x = 120 + 5n$.

$n = \frac{x-120}{5}$, price $P = 24 - 0.75n$.

Price $P(x) = 24 - 0.75\left(\frac{x-120}{5}\right)$

$P(x) = 24 - \frac{0.75x + 90}{5}$

$P(x) = 24 - \frac{0.75x}{5} + 18$

$P(x) = 120 - 0.75x + 90$

$P(x) = -0.75x + 210$

(b) The marginal revenue from weekly sales of 150 DVD's

$R(150) = -0.75x + 210$

$= -0.75(150) + 210$

$R(150) = -112.5 + 210$

$= \$97.5$ per DVD.

(c) The cost of producing x DVD's is

$C(x) = -0.003x^2 + 4.2x + 2500$ determine marginal of 150 DVDs.

$= C'(x) = 0.006x + 4.2.$

$C'(150) = 0.006 \times 150 + 4.2$

$C'(150) = 0.9 + 4.2$

$C'(150) = \$5.1$

(d) Marginal profit from the weekly sales of 150 DVDs.

$P(x) = R(x) \cdot C(x)$

$97.5 \cdot (-0.003x^2 + 4.2x + 2500)$

$P(x) = -0.2925x^2 + 409.5x + 243750$

$P'(x) = -0.585x + 409.5.$

$P'(150) = -0.585 \times 150 + 409.5$

$P'(150) = -87.7 + 409.5.$

$P'(150) = 321.75$

$P'(150) = \$ 321.75$ per DVD.

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Question 11

$h(t) = 4.9t^2 + 12t + 0.2$

(a) Height of the balloon in 2 sec

$h(2) = -4.9 \times \cancel{2\,sec} + 12 \times 2 + 0.2$

$= -9.8 + 24 + 0.2$

height $= \cancel{24m}$. $\cancel{14.4m}$. $4.6M$.

(b) Rate of change of the height at 1 sec and at 4 sec

$$\text{Rate of Change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$h(t) = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

$= f(4) = -4.9 \times 16 + 12 \times 4 + 0.2$

$= -78.4 + 48 + 0.2$

$= -30.2$.

$f(1) = -4.9 + 12 + 0.2$.

$= 7.3$.

$$\frac{-30.2 - 7.3}{3}$$

$$= \frac{-37.5}{3}$$

Rat of change $= -18.75$.

(c) How long does it take for the balloon to return to the ground.

at $h = 0$

$0 = -4.9t^2 + 12t + 0.2$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-12 \pm \sqrt{144 + 3.92}}{-9.8}$

$\dfrac{-12 \pm 12.16}{-9.8}$

$t = \dfrac{-24.16}{-9.8}$

$t = 2.4653$ sec

(d) How fast was the balloon travelling when it hit the ground.

Final velocity

$h(t) = -4.9t^2 + 12t + 0.2$

$v. = -9.8t + 12$.

Final velocity is obtain at $t = 0$

$V = -9.8 \times 0 + 12$.

$V = 12$ mps.

∴ The velocity is at maximum when $t = 0$